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MAGNETIC ANOMALIES

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INTRODUCTION

During the years 1935–37 three lines of magnetic traverse have been carried out by the Survey of India across the epicentral area of the 1934 Bihār earthquake. The results have been discussed in Survey of India Geodetic Reports 1935 and 1937. For the interpretation of such anomalies, a set of profiles is required giving the effects of various forms of magnetic deposits at different depths. This paper gives formulæ and curves showing the variations of the horizontal and vertical magnetic force corresponding to various shapes of magnetic bodies at various depths. The anomalies also depend on the magnetic dip and on the orientation of the body with respect to magnetic north, and the corresponding variations are considered. This problem has been partially considered by Dr. H. Haalck, who has given some results in "Die Magnetischen Verfahren der angewandten Geophysik", 1927, and these have been reproduced by A. B. B. Edge and T. H. Laby in "Geophysical Prospecting", 1931. The curves now given do not entirely agree with Dr. Haalck's, and it is clear that he has introduced approximations which have appreciable effects.

MAGNETIC ANOMALIES

1. Magnetic effects of different types of deposits.—Our problem is to work out the effects of the magnetism induced in different forms of rock masses by the magnetic field T of the earth. It is well-known, that when a body having greater permeability than its surroundings is placed in a magnetic field, it causes the lines of force to crowd together inside it. For weak fields like that of the earth, the magnetism induced in the body is $I=\kappa T$, where κ is the magnetic susceptibility of the body. In this treatment, we make the fundamental assumption that the magnetism induced in the body is uniform. This does not hold for crystalline bodies, which are found to be magnetically anisotropic.

The permanent magnetism of the embedded rock masses is ignored, as very little is known about it so far. We neglect also the magnetism of the body by its own lines of force. This is a reasonable assumption, since although this self-magnetism may be important for highly magnetic substances such as steel, for weakly permeable substances such as rocks, its effect will be immaterial.

Granting the above, we may employ three methods to deal with such problems :—

(a) For 2-dimensional cases, involving spheres and cylinders, spherical harmonics may be used.

(b) By the use of Poisson's Equation, the solution of a magnetic problem may be deduced from the gravitational potential of the body.

(c) Certain special cases may be solved by the use of conjugate functions, which lead to very elegant solutions.

2. Numerical values of the constants.—μ,κ denote permeability and susceptibility of the deposit. The system of units used is electromagnetic, the magnetic field being expressed in terms of $10^{-5}\,\Gamma$ or γ.

μ, κ are pure numbers. To get a quantitative idea of the anomalies produced by different types of deposits, κ is taken as 15×10^{-4}, which represents a fair average value for basalt. In deducing a structural detail from the observed magnetic anomalies, κ has to be treated as an unknown quantity. Even in the same rock type, there are enormous differences in the values of κ. It should be noted however, that by changing the value of the susceptibility, the shape of the profile is not altered. Only its vertical scale is changed in the corresponding ratio.

T represents the magnetic force, and H, V its horizontal and vertical components respectively. i denotes the dip.

The curves are drawn on the assumption T = 50,000 γ, and for values of dip equal to 0°, 20°, 40°, 60° and 90°. It would have been preferable to use the values of T appropriate to each dip, but this was not possible, as the magnetic force for the same values of the dip varies through wide limits in different parts of the globe. If the actual T differs from 50,000 γ, the anomalies given by the curves must of course be changed in simple proportion.

d denotes the thickness of the deposit, z_0 the depth to its upper surface. $d_r = d/z_0$, $x_r = x/z_0$.

In the case of the spherical and the cylindrical deposits, z_0 denotes the depths of their centres below the ground.

β is the azimuth of the axes of two-dimensional features. The curves are drawn for $\beta = 0°$, 45°, and 90°, the line of traverse being taken perpendicular to the feature. F, Z denote the components of the magnetic anomaly along the traverse line and in the vertical direction. The positive direction of F is in each case shown in the diagram.

3. Anomalies due to a sphere of susceptibility κ.—Suppose in the first instance that the sphere is placed in a field of uniform magnetic force parallel to the x-axis.

Let V_0 represent the potential external to the sphere, and V_1 the potential inside it. At a great distance,

$$V_0 = -Hx = -HrP_1$$

$$\left.\begin{aligned} \text{Hence, } V_0 \text{ at any point} &= -HrP_1 + \frac{AP_1}{r^2}, \\ \text{and} \quad V_1 &= BrP_1 \end{aligned}\right\}$$

The two constants A and B are easily obtained from the boundary conditions

$$V_0 = V_1 \quad \text{at } r = a$$

$$\frac{\delta V_0}{\delta r} = \mu \frac{\delta V_1}{\delta r} \quad \text{at } r = a$$

If R and S represent the radial and transverse components of the magnetic force at a point, we have

$$R = -\frac{\delta V_0}{\delta r}, \; S = -\frac{\delta V_0}{r\,\delta\theta}.$$

$$\left.\begin{aligned} \text{Obviously } Z &= -S\cos\theta - R\sin\theta = -H\frac{3a^3}{2r^3}\cdot\frac{4\pi\kappa}{3+4\pi\kappa}\sin 2\theta. \\ F &= -S\sin\theta + R\cos\theta = H\frac{a^3}{r^3}\cdot\frac{4\pi\kappa}{3+4\pi\kappa}(2\cos^2\theta - \sin^2\theta). \end{aligned}\right\} \quad \ldots (1)$$

Hence the anomalies due to a sphere placed in a field of potential $\Omega = -Hx - Vz$ are

$$\left.\begin{aligned} Z &= \frac{a^3}{r^5}\cdot\frac{4\pi\kappa H}{3+4\pi\kappa}\left[(2z_0^2 - x^2)\tan i - 3xz_0\right]. \\ F &= \frac{a^3}{r^5}\cdot\frac{4\pi\kappa H}{3+4\pi\kappa}\left[(2x^2 - z_0^2) - 3xz_0\tan i\right]. \end{aligned}\right\} \quad \ldots\ldots (2)$$

The variations of Z and F are shown in Plate I for the case $a/z_0 = 1$, i.e., when the sphere touches the ground level. The chart can however be used for other values of a/z_0 by diminishing the vertical scale as the cube of this ratio. We notice, that the gradients near $x = 0$ are rather steep. For points beyond the periphery of the sphere, the values of F, Z quickly converge to zero, indicating that a spherical deposit has no far reaching effects.

4. An infinite circular cylinder, placed at right angles to the earth's field.—The cylinder is placed with its axis along the axis of y, the x-axis being chosen along the magnetic meridian. We will consider first the effect of the magnetism induced by the component H of the earth's magnetic field. Before the introduction of the cylinder, the field is $V = -Hr\cos\theta$.

After the introduction of the cylinder, let the field be

$$\left.\begin{aligned} V_0 &= -Hr\cos\theta + \Sigma\frac{A_n\cos n\theta}{r^n} \\ V_1 &= \Sigma C_n r^n \cos n\theta. \end{aligned}\right\}$$

Applying the normal boundary conditions and proceeding as above, we get

$$\left.\begin{aligned} Z_1 &= -H.\frac{8\pi\kappa a^2}{2+4\pi\kappa}\cdot\frac{\sin\theta\cos\theta}{r^2} \\ F_1 &= H.\frac{4\pi\kappa a^2}{2+4\pi\kappa}\cdot\frac{\cos^2\theta - \sin^2\theta}{r^2} \end{aligned}\right\} \quad \ldots\ldots\ldots (3)$$

The anomalies due to a cylinder placed in a magnetic field of potential $V = -(Hx + H \tan i \,.\, y)$ are therefore

$$Z = \frac{4\pi\kappa H}{2+4\pi\kappa} \cdot \frac{a^2}{r^4} \left[-2z_0 x + (z_0^2 - x^2) \tan i \right]$$

$$F = \frac{4\pi\kappa H}{2+4\pi\kappa} \cdot \frac{a^2}{r^4} \left[(x^2 - z_0^2) - 2xz_0 \tan i \right]$$

These anomalies are shown in Plates I and II.

The curves for spherical and cylindrical deposits are practically identical in character. The difference is mainly in the absolute values and in the fact that the decrement with depth is less rapid in the case of a cylindrical deposit. The vertical scale has to be reduced in the ratio $(a/z_0)^2$. As an example, Figs. 1 and 3 of Plate I show that for $i = o$, $\beta = o$, the values of F for the two deposits at the origin are -313γ and -470γ respectively for $a/z_0 = 1$. If the depth be increased three times, the value for the sphere reduces to -11γ, while that for the cylinder is -52γ.

5. An infinite elliptic cylinder.—Consider next the case of an infinite elliptic cylinder $x^2/a^2 + z^2/b^2 = 1$, placed in the earth's magnetic field. As before, we will first find the effect of the component H.

Put $x + iz = c \cosh(\xi + i\eta)$

$x = c \cosh\xi \cos\eta$, $z = c \sinh\xi \sin\eta$;

and
$$\frac{x^2}{c^2\cosh^2\xi} + \frac{z^2}{c^2\sinh^2\xi} = 1$$

$a = c\cosh\alpha$, $b = c\sinh\alpha$ define the semi-axes of the ellipse.

$\xi = \alpha$ represents the elliptic cylinder. Undisturbed $V = -Hx = -Hc\cosh\xi\cos\eta$.

Let $\Omega_1 = A\cosh\xi\cosh\eta$,

$\Omega_0 = -Hc\cosh\xi\cos\eta + Be^{-\xi}\cos\eta$

At $\xi = \alpha$, $\Omega_0 = \Omega_1$, and $\mu\dfrac{\delta\Omega_1}{\delta\xi} = \dfrac{\delta\Omega_0}{\delta\xi}$.

Hence
$$A\cosh\alpha = -Hc\cosh\alpha + Be^{-\alpha}$$
$$\mu\, A\sinh\alpha = -Hc\sinh\alpha - Be^{-\alpha}$$

$$A(\cosh\alpha + \mu\sinh\alpha) = -Hc(\cosh\alpha + \sinh\alpha) \text{ or } A = \frac{-Hc\,(e^\alpha)}{\cosh\alpha + \mu\sinh\alpha}$$

$$B = \left[Hc\cosh\alpha - \frac{Hc\cosh\alpha \,.\, e^\alpha}{\cosh\alpha \sinh\alpha} \right] e^\alpha$$

$$= Hc\,e^\alpha\cosh\alpha \left[\frac{\mu\sinh\alpha - \sinh\alpha}{\cosh\alpha + \mu\sinh\alpha} \right]$$

$$= \frac{Hc\,e^\alpha\cosh\alpha\sinh\alpha\,(\mu - 1)}{\cosh\alpha + \mu\sinh\alpha}$$

$$F = \frac{\delta}{\delta x}(Be^{-\xi}\cos\eta),\quad Z = -\frac{\delta}{\delta y}(Be^{-\xi}\cos\eta).$$

Now
$$F = -\frac{\delta}{\delta x}(Be^{-\xi}\cos\eta)$$

$$= Be^{-\xi}\cos\eta\frac{\delta\xi}{\delta x} + Be^{-\xi}\sin\eta\frac{\delta\eta}{\delta x}$$

$$x^2 \operatorname{sech}^2\xi + y^2 \operatorname{cosech}^2\xi = c^2$$
$$x^2 \sec^2\eta - y^2 \operatorname{cosec}^2\eta = c^2$$

whence

$$\frac{d\xi}{dx} = \frac{x}{x^2 \tanh\xi + y^2 \coth^3\xi}$$

and $$\frac{d\eta}{dx} = -\frac{x}{x^2 \tan\eta + y^2 \cot^3\eta}$$

so that

$$F = Be^{-\xi}\frac{x\cos\eta}{x^2 \tanh\xi + y^2 \coth^3\xi} - Be^{-\xi}\frac{x\sin\eta}{x^2 \tan\eta + y^2 \cot^3\eta}$$

similarly

$$Z = Be^{-\xi}y\left[\frac{\cos\eta}{x^2 \tanh^3\xi + y^2 \coth\xi} + \frac{\sin\eta}{x^2 \tan^3\eta + y^2 \cot\eta}\right] \qquad (4)$$

For a circular cylinder $c\cosh\xi = c\sinh\xi = r$, $c\cosh a = c\sinh a = a$

$$\therefore\ F = Be^{-\xi}\frac{x\cos\eta}{x^2+y^2} - \frac{Be^{-\xi}x\sin\eta}{\frac{x^2\sin\eta}{\cos\eta} + \frac{y^2\cos^3\eta}{\sin^3\eta}}$$

$$= Be^{-\xi}\frac{x\cos\eta}{x^2+y^2} - Be^{-\xi}\frac{x\sin^2\eta}{r^2\cos\eta}$$

$$= Be^{-\xi}\left[\frac{\cos^2\eta}{r} - \frac{\sin^2\eta}{r}\right]$$

$$= \frac{Ha(\mu-1)}{\mu+1}e^{a-\xi}\left(\frac{\cos^2\eta - \sin^2\eta}{r}\right)$$

$$e^{a-\xi} = \frac{ce^a}{ce^\xi} = \frac{c(\cosh a + \sinh a)}{c(\cosh\xi + \sinh\xi)} = \frac{a}{r}$$

$$\therefore\ F = \frac{Ha^2(\mu-1)}{\mu+1}\cdot\frac{(\cos^2\eta - \sin^2\eta)}{r^2} \qquad (5)$$

This agrees with formula (3) obtained above. Formula (5) can easily be extended to the case when the cylinder is placed in a field of potential $V = -(Xx + Yy)$.

6. Inclined block of thickness 'd':—Suppose a block* of thickness d is inclined to the horizontal at an angle a and suppose the block extends to infinity downwards, and on both sides perpendicular to the plane of the figure. We will give a complete proof of this case as it is of great practical importance.

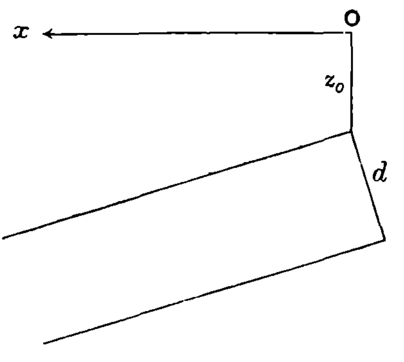

Fig. 1.

* This case has been treated by Haalck on pp. 55–60, but all his formulæ appear to be incorrect.

Poisson's Law states that the potential of a magnetic element dv (x, y, z) at a point P (ξ, η, ζ) is given by

$$W = \iiint dv \left\{ A\,\frac{\xi - x}{r^3} + B\,\frac{\eta - y}{r^3} + C\,\frac{\zeta - z}{r^3} \right\}.$$

In our case $A = \kappa H$, $B = 0$ and $C = \kappa Z$. The potential at O due to an elementary volume at (x', y', z') is therefore

$$\left(\kappa H\,\frac{x_0 - x'}{r^3} - \kappa V\,\frac{z'}{r^3} \right) dx'dy'dz',$$

and the potential of the whole mass is

$$W = \kappa \iiint \frac{[H(x_0 - x') - Vz']}{r^3}\, dx'dy'dz'. \qquad (6)$$

Integrating first of all with respect to y' we get the contribution of the infinite strip of cross-section $dx'dz'$. Along this section x' and z' are constant as a little consideration will show, and the integration with respect to y' is therefore justifiable. Limits of y' are $-\infty$ and $+\infty$.

We get $$W = 2\kappa \iint \frac{H(x_0 - x') - z'V}{(x_0 - x')^2 + z'^2}\, dx'dz' \qquad (7)$$

Choose a new system of axes x'', z'' as shown.

Consider a point P whose old and new co-ordinates are (x', z') and (x'', z'') respectively.

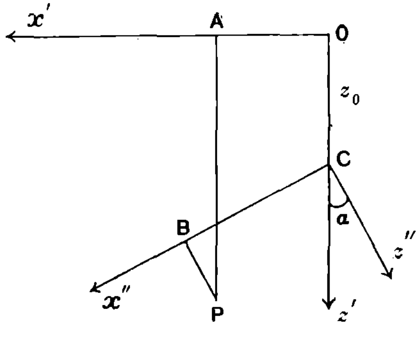

Fig. 2

Obviously OA = Projection of $CB + BP$.

$$\left. \begin{aligned} &\text{or } x' = x'' \cos a - z'' \sin a. \\ &\text{Similarly } \quad z' = z_0 + x'' \sin a + z'' \cos a. \end{aligned} \right\} \qquad (8)$$

$$W = 2\kappa \iint \frac{H(x_0 - x'' \cos a + z'' \sin a) - V(z_0 + x'' \sin a + z'' \cos a)}{(x_0 - x'' \cos a + z'' \sin a)^2 + (z_0 + x'' \sin a + z'' \cos a)^2}\, dx''dz''$$

$$= 2\kappa \iint \frac{(Hx_0 - Vz_0) - x''(H \cos a + V \sin a) + z''(H \sin a - V \cos a)}{(x'' + z_0 \sin a - x_0 \cos a)^2 + (z'' + z_0 \cos a + x_0 \sin a)^2}\, dx''dz'' \qquad (9)$$

To integrate with respect to z'', put $z''' = z'' + z_0 \cos a + x_0 \sin a$,

$A = (Hx_0 - Vz_0) - x''(H \cos a + V \sin a) - (z_0 \cos a + x_0 \sin a)(H \sin a - V \cos a)$

$B = (H \sin a - V \cos a)$.

$c = x'' + z_0 \sin a - x_0 \cos a$.

Then $W = 2\kappa \iint \frac{A + B z'''}{z'''^2 + c^2} dx''\, dz'''$

$$= 2\kappa \int \left[\frac{A}{c} \tan^{-1} \frac{z'''}{c} + \frac{B}{2} \log\ (z'''^2 + c^2) \right] dx''$$

$$= 2\kappa \int \left[\left\{ \frac{ (H x_0 - V z_0) - (z_0 \cos a + x_0 \sin a) (H \sin a - V \cos a) - x'' (H \cos a + V \sin a) }{ x'' + z_0 \sin a - x_0 \cos a } \times \right.\right.$$

$$\left. \tan^{-1} \frac{z'''}{x'' + z_0 \sin a - x_0 \cos a} \right\} + \frac{H \sin a - V \cos a}{2} \times$$

$$\left. \log \left\{ z'''^2 + (x'' + z_0 \sin a - x_0 \cos a)^2 \right\} \right] dx'' \quad . \; . \; . \; . \quad (10)$$

Putting $x''' = x'' + z_0 \sin a - x_0 \cos a$

$$W = 2\kappa \int - (H \cos a + V \sin a) \tan^{-1} \frac{z'''}{x'''} dx'''$$

$$+ 2\kappa \int \frac{H \sin a - V \cos a}{2} \log\ (z'''^2 + x'''^2)\ dx'''$$

Integrating with respect to x'''

$$W = 2\kappa \left[- (H \cos a + V \sin a) \left\{ x''' \tan^{-1} \frac{z'''}{x'''} + \frac{z'''}{2} \log\ (z'''^2 + x'''^2) \right\} \right.$$

$$\left. + \frac{H \sin a - V \cos a}{2} \left\{ x''' \log (z'''^2 + x'''^2) - 2 x''' + 2 z''' \tan^{-1} \frac{x'''}{z'''} \right\} \right]$$

$$. \; . \; . \; . \; . \; . \quad (11)$$

Now $\quad z''' = z'' + z_0 \cos a + x_0 \sin a$

and $\quad z''$ varies from 0 to z,

therefore $\quad z'''$ varies from $(z_0 \cos a + x_0 \sin a)$ to $(z + z_0 \cos a + x_0 \sin a)$

Evaluating between these limits

$$\frac{W}{2\kappa} = - (H \cos a + V \sin a) \left[\left\{ x''' \tan^{-1} \frac{z + z_0 \cos a + x_0 \sin a}{x'''} \right.\right.$$

$$\left. - x''' \tan^{-1} \frac{z_0 \cos a + x_0 \sin a}{x'''} \right\} + \left\{ \frac{z + z_0 \cos a + x_0 \sin a}{2} \times \right.$$

$$\log\ [(z + z_0 \cos a + x_0 \sin a)^2 + x'''^2] - \frac{z_0 \cos a + x_0 \sin a}{2} \times$$

$$\left.\left. \log\ [(z_0 \cos a + x_0 \sin a)^2 + x'''^2] \right\} \right]$$

$$+ \frac{H \sin a - V \cos a}{2} \left[x''' \log \frac{x'''^2 + (z + z_0 \cos a + x_0 \sin a)^2}{x'''^2 + (z_0 \cos a + x_0 \sin a)^2} \right.$$

$$+ 2\ (z + z_0 \cos a + x_0 \sin a) \tan^{-1} \frac{x'''}{z + z_0 \cos a + x_0 \sin a}$$

$$\left. - 2\ (z_0 \cos a + x_0 \sin a) \tan^{-1} \frac{x'''}{z_0 \cos a + x_0 \sin a} \right] \quad . \; . \; . \; . \; . \; . \quad (12)$$

Now $\quad x''$ varies from 0 to ∞,

therefore $\quad x'''$ varies from $(z_0 \sin a - x_0 \cos a)$ to ∞.

$$\text{and}\ \frac{W}{2\kappa} = -(H\cos a + V\sin a)\left[\left\{z - (z_0\sin a - x_0\cos a)\tan^{-1}\frac{z+z_0\cos a + x_0\sin a}{z_0\sin a - x_0\cos a}\right.\right.$$

$$\left. + (z_0\sin a - x_0\cos a)\tan^{-1}\frac{z_0\cos a + x_0\sin a}{z_0\sin a - x_0\cos a}\right\} - \frac{z+z_0\cos a + x_0\sin a}{2}\times$$

$$\log\left\{(z+z_0\cos a + x_0\sin a)^2 + (z_0\sin a - x_0\cos a)^2\right\} + \frac{z_0\cos a + x_0\sin a}{2}\times$$

$$\left.\log\left\{(z_0\cos a + x_0\sin a)^2 + (z_0\sin a - x_0\cos a)^2\right\}\right] - \frac{H\sin a - V\cos a}{2}\times$$

$$\left[-\pi z + (z_0\sin a - x_0\cos a)\log\frac{(z_0\sin a - x_0\cos a)^2 + (z+z_0\cos a + x_0\sin a)^2}{x_0^2+z_0^2}\right.$$

$$+2(z+z_0\cos a + x_0\sin a)\tan^{-1}\frac{z_0\sin a - x_0\cos a}{z+z_0\cos a + x_0\sin a}$$

$$\left.-2(z_0\cos a + x_0\sin a)\tan^{-1}\frac{z_0\sin a - x_0\cos a}{z_0\cos a + x_0\sin a}\right] \quad \ldots\ldots\ldots \quad (13)$$

Differentiating this expression with respect to x_0 we have

$$-\frac{F}{2\kappa} = -(H\cos a + V\sin a)\cos a\left\{\tan^{-1}\frac{z+z_0\cos a + x_0\sin a}{z_0\sin a - x_0\cos a} - \tan^{-1}\frac{z_0\cos a + x_0\sin a}{z_0\sin a - x_0\cos a}\right\}$$

$$-(H\sin a - V\cos a)\sin a\left\{\tan^{-1}\frac{z_0\sin a - x_0\cos a}{z+z_0\cos a + x_0\sin a} - \tan^{-1}\frac{z_0\sin a - x_0\cos a}{z_0\cos a + x_0\sin a}\right\}$$

$$+\frac{(H\cos a + V\sin a)\sin a}{2}\left\{\log\frac{(z+z_0\cos a + x_0\sin a)^2 + (z_0\sin a - x_0\cos a)^2}{(z_0\cos a + x_0\sin a)^2 + (z_0\sin a - x_0\cos a)^2}\right\}$$

$$+\frac{(H\sin a - V\cos a)\cos a}{2}\left\{\log\frac{(z+z_0\cos a + x_0\sin a)^2 + (z_0\sin a - x_0\cos a)^2}{(z_0\cos a + x_0\sin a)^2 + (z_0\sin a - x_0\cos a)^2}\right\}$$

$$\ldots\ldots \quad (14)$$

A similar expression holds for Z.

Putting $z=d$ and $a=o$ in equation (13) we obtain the potential due to a horizontal slab of thickness d.

$$\frac{W}{2\kappa} = -H\left[d + x_0\left(\tan^{-1}\frac{z_0}{x_0} - \tan^{-1}\frac{d+z_0}{x_0}\right) + \frac{z_0}{2}\log(z_0^2+x_0^2)\right.$$

$$\left.-\frac{d+z_0}{2}\log\left\{(d+z_0)^2 + x_0^2\right\}\right] + \frac{V}{2}\left[-\pi d - x_0\log\frac{x_0^2 + (d+z_0)^2}{x_0^2+z_0^2}\right.$$

$$\left.+2z_0\tan^{-1}\frac{x_0}{z_0} - 2(d+z_0)\tan^{-1}\frac{x_0}{d+z_0}\right] \quad \ldots\ldots\ldots\ldots \quad (15)$$

$$\frac{F}{2\kappa} = -\frac{1}{2\kappa}\frac{\delta W}{\delta x_0}$$

$$= H\left[\tan^{-1}\frac{z_0}{x_0} - \tan^{-1}\frac{d+z_0}{x_0}\right] + \frac{V}{2}\log\frac{x_0^2+(d+z_0)^2}{x_0^2+z_0^2} \quad \ldots\ldots \quad (16)^*$$

$$\frac{Z}{2\kappa} = -\frac{1}{2\kappa}\frac{\delta W}{\delta z_0}$$

$$= \frac{H}{2}\log\frac{z_0^2+x_0^2}{(z_0+d)^2+x_0^2} - \frac{V}{2}\left(2\tan^{-1}\frac{x_0}{z_0} - 2\tan^{-1}\frac{x_0}{d+z_0}\right) \quad \ldots\ldots \quad (17)^*$$

* These formulæ are different from those obtained by Haalck, although an identical method has been used in their derivation. Haalck's evaluation of the expression for the magnetic potential Γ on page 56 of his book is incorrect.

We see that the working of these 2-dimensional problems with the help of the gravitational potential is very laborious. The computations become much simpler, if the effect of each face of the magnetic body is computed separately. Consider a plane of width b, extending infinitely in the direction perpendicular to the plane of the paper, the intensity of magnetization being I per unit length. The attraction of the infinite strip of width dx at C is $2\,I\,dx/r$ along PC. Resolving along the horizontal and vertical directions, and integrating throughout the total breadth of the plane, we obtain the very convenient expressions

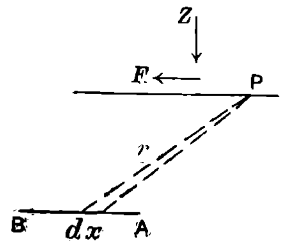

Fig. 3

$$Z = 2\,I\,\phi, \qquad F = 2\,I \log r_2/r_1, \text{ where } \phi = \angle \text{APB}.$$

Applying these formulæ to the boundaries of Fig. 1, we get without difficulty the expressions for F and Z.

This method is justifiable, because by applying Green's Theorem to Poisson's Equation, we see that for a uniformly magnetized body the integration can be reduced to a surface integral along the boundary of the magnetic body, the surface density being $I \cos \theta$, where θ is the angle between the direction of magnetization and the normal to the body. The volume density can be taken to be nil.

7. Simple geological fault, extending to a great depth.—A rectangular slab, infinite in one linear direction, and also of infinite extent in both directions perpendicular to the plane of the paper. This is equivalent to a simple geological fault, extending to a great depth since the continuous magnetic material below CD in Fig. 4 will cause no anomaly.

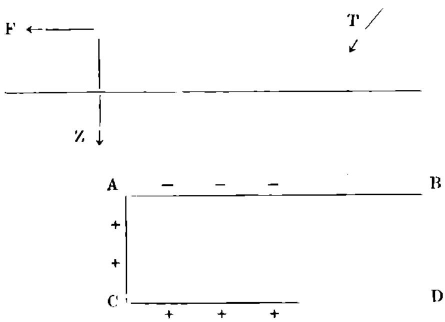

Fig. 4.

From the preceding paragraph, we see that

$$F_{AB} + F_{CD} = -2\kappa\, V \log r_C/r_A$$

$$F_{AC} = +2\kappa\, H\, (\phi_{AC})$$

$$Z_{AC} = -2\kappa\, H \log r_C/r_A$$

$$Z_{AB} + Z_{CD} = -2\kappa\, V\, (\phi_{AC})$$

Hence

$$\left.\begin{aligned} F &= -\kappa H \tan i \log_e \frac{x^2+(z_0+d)^2}{x^2+z_0^2} + 2k\,H\left(\tan^{-1}\frac{x}{z_0} - \tan^{-1}\frac{x}{z_0+d}\right)\cos\beta \\ &= -\kappa H \tan i \log_e \frac{x_r^2+(1+d_r)^2}{x_r^2+1} + 2k\,H\left(\tan^{-1}x_r - \tan^{-1}\frac{x_r}{1+d_r}\right)\cos\beta \\ Z &= -\kappa H \cos\beta \log_e \frac{x_r^2+(1+d_r)^2}{x_r^2+1} - 2k\,H \tan i\left(\tan^{-1}x_r - \tan^{-1}\frac{x_r}{1+d_r}\right) \end{aligned}\right\} \quad . \quad . \; (19)$$

Plates III and IV exhibit the curves for $d_r = 20$, and 1000.

Since the effect of an infinite plate of finite thickness, having opposite polarity on its two faces is nil, we see that the above formulæ also hold for a deposit of the form shown in Fig. 5.

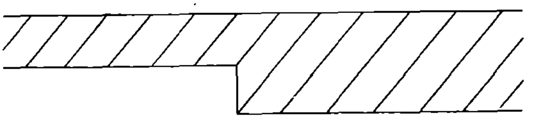

Fig. 5.

8. Simple geological fault, extending to a finite depth.—This case is easily deducible from the formulæ in para 7.

The results for some concrete cases are shown in Plates V and VI. The deposits in Plate VI are not really simple faults, but can be reduced to them by the addition of a rectangular slab.

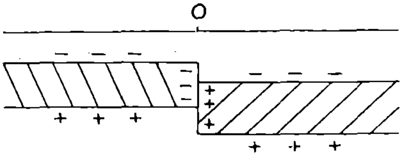

Fig. 6.

9. Inclined slab.—A slab, extending infinitely in both directions perpendicular to the plane of the paper, with one face sloping at an angle a.

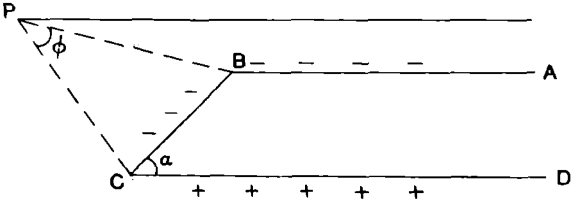

Fig. 7.

$$F_{AB} + F_{CD} = -2\,I \log r_C/r_B$$

$$Z_{AB} + Z_{CD} = -2\,I\,\phi_{BC}$$

$$= -2\,I\left[\tan^{-1}\frac{z_0+d}{x-d\cot a} - \tan^{-1}\frac{z_0}{x}\right]$$

$$F = -2\kappa\,V \log r_C/r_B + 2\kappa\,T \sin(40-a)\cos a \log r_C/r_B$$
$$- 2\kappa\,T \sin a \sin(40-a)\,[\,\phi\,].$$

$$\left.\begin{aligned}
\text{or } \frac{F}{2\kappa T} &= -\left[\log \sqrt{\frac{(x-d\cot a)^2+(z_0+d)^2}{\sqrt{x^2+z_0^2}}}\right][\sin i-\sin(i-a)\cos a] \\
&\qquad -\sin a \sin \overline{i-a}\,[\phi]. \\
&= -[\sin i-\sin\overline{i-a}\cos a]\left[\log\sqrt{\frac{(x_r-d_r\cot a)^2+(1+d_r)^2}{\sqrt{1+x_r^2}}}\right] \\
&\qquad -\sin a \sin(40-a)\,[\phi]. \\
\frac{Z}{2\kappa T} &= \sin(i-a)\sin a \log \frac{\sqrt{(x_r-d_r\cot a)^2+(1+d_r)^2}}{\sqrt{1+x_r^2}} \\
&\qquad -[\sin i-\sin\overline{i-a}\cos a]\,[\phi].
\end{aligned}\right\} \quad (20)$$

where $[\phi] = \tan^{-1} x_r - \tan^{-1}\dfrac{x_r-d_r\cot a}{1+d_r}$

Plate VII gives the anomalies for different values of a for $i=20^\circ$ and $d_r=20$.

It also shows the effect of varying the azimuth.

Plate VIII exhibits the anomalies for $a=2^\circ$, $d_r=20$, $\beta=0$ for different values of the dip.

10. Vertical Dyke.—Vertical dyke with breadth $2b$, having an infinite vertical extent.

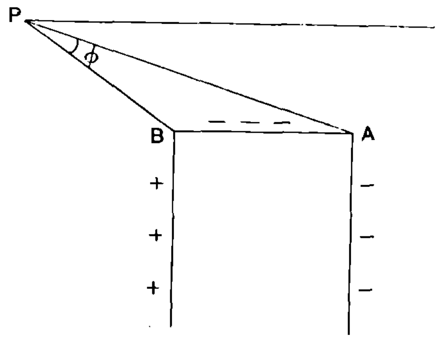

Fig. 8.

$$\left.\begin{aligned}
F &= 2\kappa H \tan i \log_e r_B/r_A - 2\kappa H\,[\phi]\cos\beta \\
Z &= 2\kappa H \tan i\,[\phi] + 2\kappa H \log_e r_B/r_A \cos\beta
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (21)$$

These formulæ also hold for a deposit of the form shown in Fig. 9.

The anomalies due to such a deposit are shown in Plates VIII and IX for $b_r=1$, and $b_r=1000$.

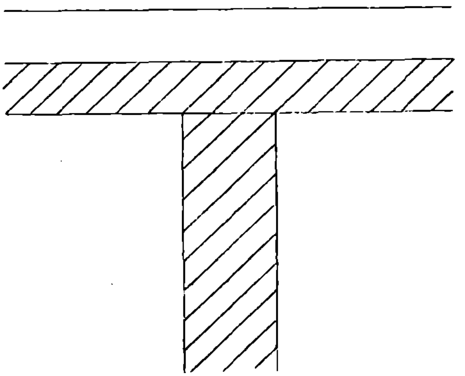

Fig. 9.

11. Block of thickness d, width $2b$.—We will next consider a block of thickness d, depth z_0, width $2b$, extending infinitely in both directions perpendicular to the plane of paper.

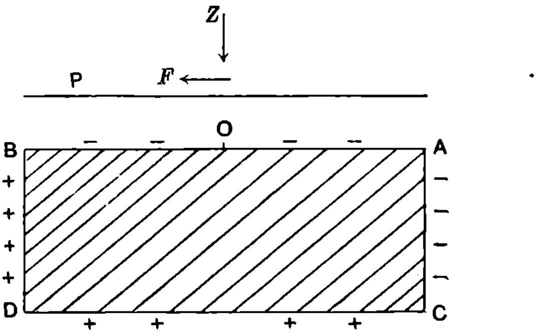

Fig. 10.

$$\left.\begin{aligned} F &= -2\kappa\, H \tan i \log_e \frac{r_A\, r_D}{r_B\, r_C} + 2\kappa\, H\,(\angle BPD - \angle APC)\cos\beta \\ Z_0 &= -2\kappa\, H \cos\beta \log_e \frac{r_A\, r_D}{r_B\, r_C} - 2\kappa\, H \tan i\,(\angle CPD - \angle APB) \end{aligned}\right\} \quad \ldots \quad (22)$$

Plate X shows the anomalies for $d_r = 10$, $b_r = 10$ for different values of dip and Plate XI for $d_r = 1000$, $b_r = 1000$ and $d_r = 100$, $b_r = 100$, for dip $= 0°$ and $40°$.

12. Use of conjugate functions.—We will finally give an example of the use of conjugate functions, by which we can get solution of one magnetic problem from another known one by a transformation of the type $u + iv = f(x + iy)$. Uhrig and Schafer* have solved the following problem by this method. An infinite cylinder of permeability μ has a sinusoidal boundary as given by $y = \cos^{-1} \dfrac{e^{-b} - e^{2x}}{e^{x}(1 - e^{-b})}$. It is placed in a uniform field τ inclined at an angle a to it. Find Z and F due to the induced magnetism.

The transformation needed to solve this problem is $w = e^z$. It might be remarked however that for all practical purposes bearing in mind the uncertainty inherent in magnetic methods, the cases 1 to 11 dealt with above, should suffice. With sufficient approximation, the sinusoidal boundary in the above case may be represented by plane faces *AC, BD.* The effect of the body as shown in Fig. 11 is easily calculable by the above formulæ.

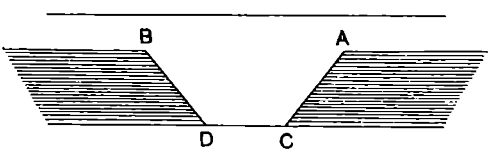

Fig. 11.

13. Practical use of the curves.—The curves of this paper can be put to several uses. From them, we can obtain either the effects of varying the thickness of a deposit of given depth, or of varying the depth keeping the thickness fixed. As an example, consider the case of a simple horizontal slab. Suppose, we are given that at a place, where dip is 20°, the thickness of the deposit is $\frac{1}{4}$ mile, and we want to compare its effects, when placed at depths $\frac{1}{40}$ mile, $\frac{1}{20}$ mile and $\frac{1}{2}$ mile respectively. Plate III, Figs. 3 and 4 yield this information easily. As z_0 changes, the horizontal scale alters in the corresponding ratio.

* Uhrig and Schafer, "Gerl. Beit. Zur Geophysik". Band 49, p. 129.

For example in Fig. 4, if $z_0 = \frac{1}{2}$ mile, the horizontal scale is 1 mile $= 0 \cdot 1$ inch.

For $z_0 = \frac{1}{20}$ mile, the scale becomes $\frac{1}{20}$ mile $= 0 \cdot 1$ inch, and so on.

If z_0 is fixed and d only is changed, then the horizontal scale remains unaltered. Thus, let $z_0 = 1$ mile and $d = 1$, 10, and 20 miles. The horizontal scale in Fig. 4 is 1 mile $= \frac{1}{20}$ inch in all the three cases. We can see at a glance, how the maximum values of F, Z increase with thickness.

The curves also enable one to judge the effect of different orientations.

Formulæ on similar lines can also be developed for the case when the two-dimensional deposit is not infinite in both directions perpendicular to the paper. As an example consider the effect of a vertical step, which extends to infinity in the positive direction of the y-axis. Suppose the line of traverse is at a distance c along the y-axis from its finite end. The effect of such a block for various values of c can be worked out, although the integrations become rather cumbrous. It might be noted however that in this case, there will be a horizontal component of disturbing force both in the direction of the x-axis, as well as in the direction of the y-axis.

We will give the formulæ for the case of a plate of finite breadth b, as shown in Fig. 12.

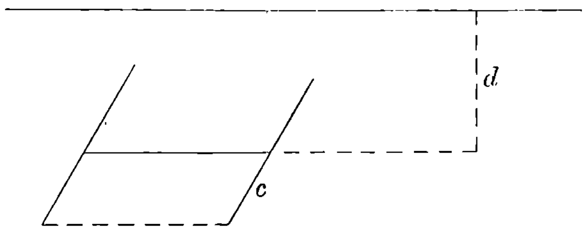

Fig. 12.

$$Z = \int_{x_1}^{x_2} \frac{I\,d}{x^2 + d^2}\,dx + \int_{x_1}^{x_2} \frac{I\,c\,d}{(x^2 + d^2)\sqrt{c^2 + x^2 + d^2}}\,dx$$

$$F_y = \int_{x_1}^{x_2} \frac{I}{\sqrt{x^2 + c^2 + d^2}}\,dx$$

$$F_x = \int_{x_1}^{x_2} \frac{I\,x}{x^2 + d^2}\,dx + \int_{x_1}^{x_2} \frac{I\,c\,x}{\sqrt{x^2 + d^2}\sqrt{c^2 + x^2 + d^2}}\,dx$$

These are all standard forms of integrals, which can be evaluated without difficulty.

The application of these formulæ to the cases discussed in paras 1 to 11 is obvious. The formulæ however become rather complicated.

14. General remarks.—The magnetic method for locating embedded deposits is burdened with many uncertainties, as magnetism is very liable to small changes in the chemical composition of a substance. It is well-known that rocks of the same type show considerable variations from one specimen to another. The presence of permanent magnetism may also obscure the problem. Conclusions can however be made more reliable if the body producing the anomaly is close to the surface, so that samples can be collected, and tested for permanent magnetism and changes of susceptibility, which can then be allowed for.

It will be seldom that magnetic survey or any other geophysical method can by itself lead to definite conclusions regarding concealed structures, but each different method (magnetic, seismic or gravimetric) provides evidence of a different kind, and a combination of the different lines of evidence will often lead to conclusive results.

15. Summary.—Formulæ for magnetic anomalies produced by induced magnetism in different forms of magnetic deposits are given. The variations of these anomalies for traverses along different azimuths have been shown graphically for different depths of the deposits, taking the numerical value of the earth's magnetic force as 50,000γ.

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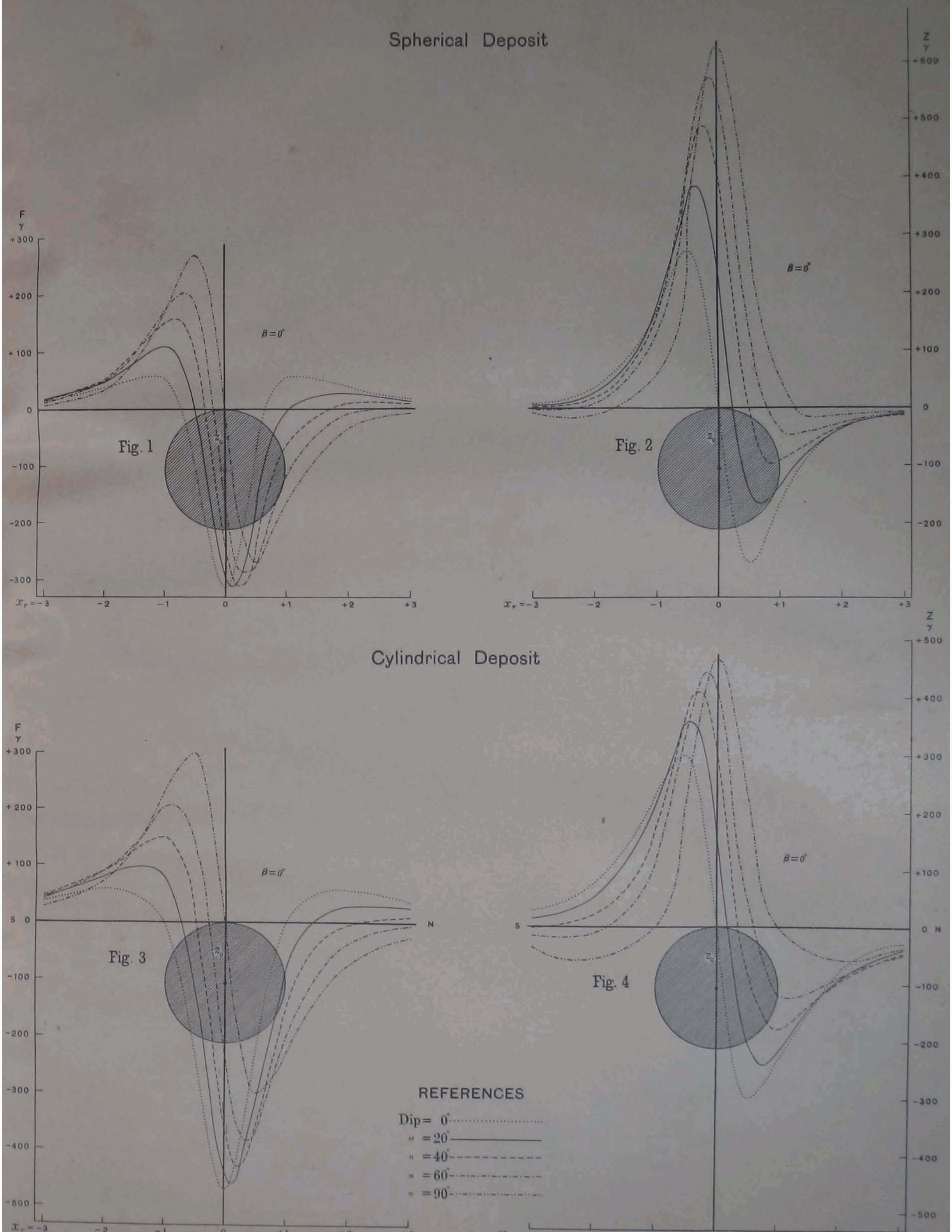
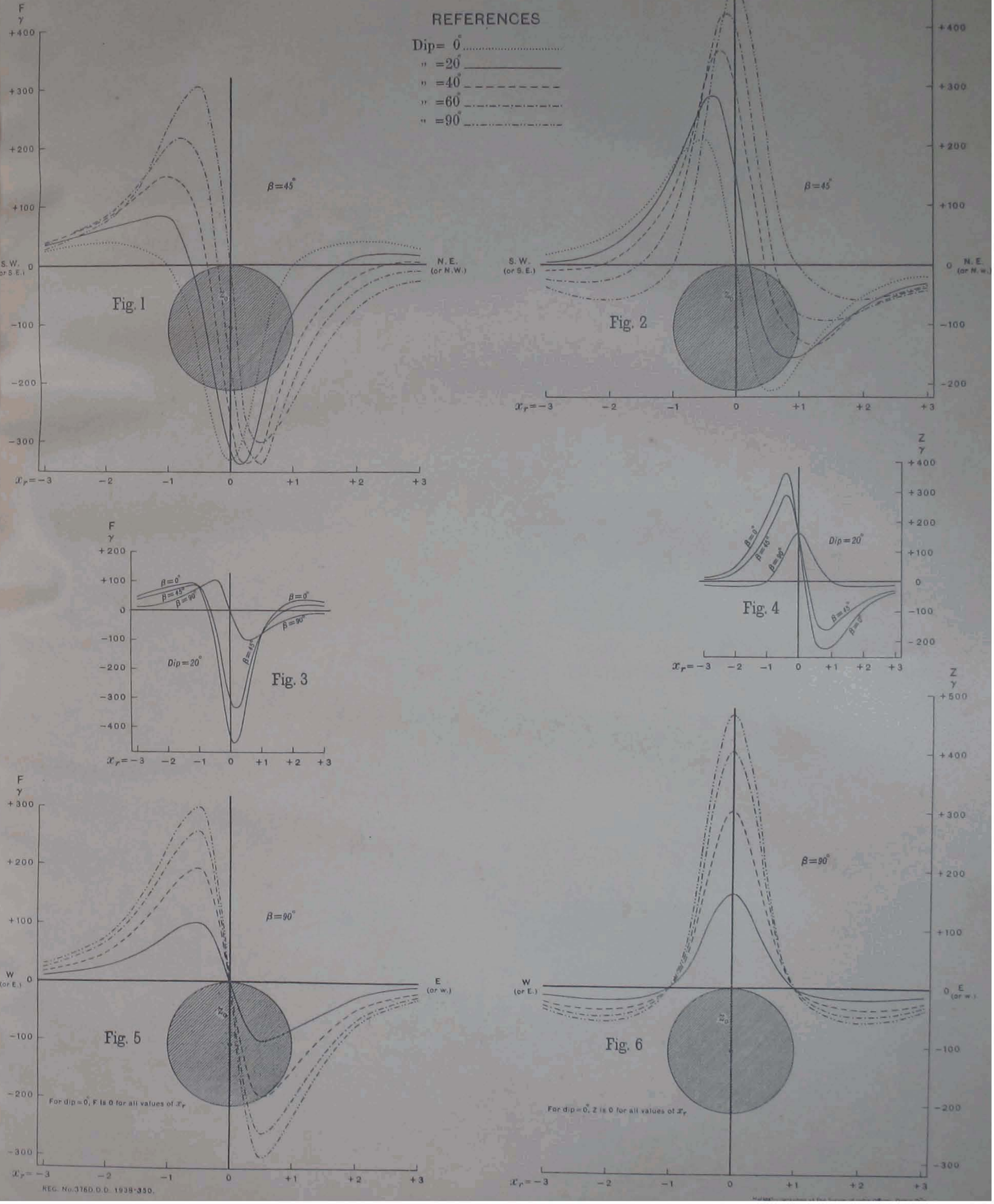
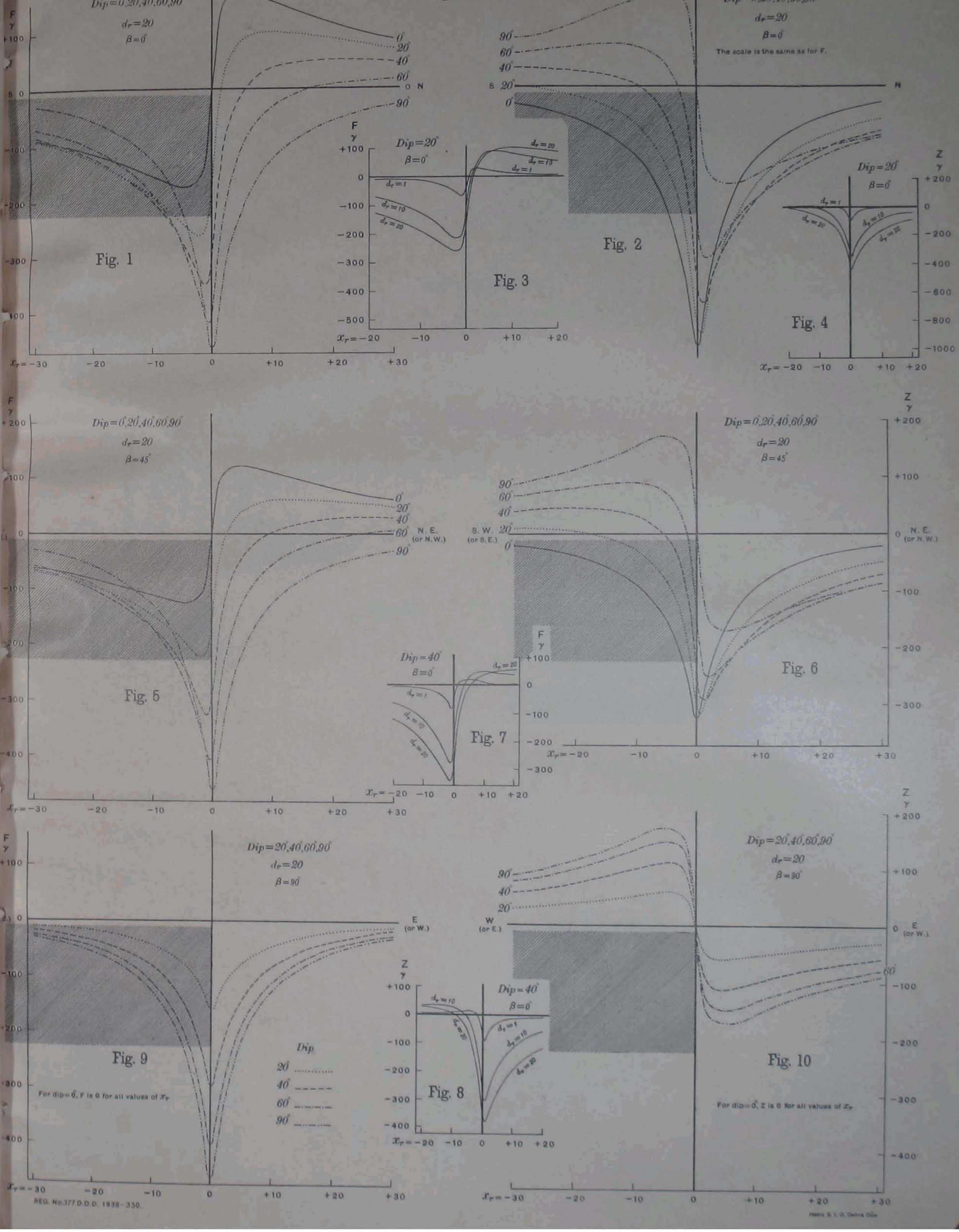

Spherical Deposit
Fig. 1
Fig. 2
β = 0°
Cylindrical Deposit
Fig. 3
Fig. 4
β = 0°
REFERENCES
Dip = 0°
" = 20°
" = 40°
" = 60°
" = 90°
REG. No. 2750 D.D. 1936-350.

Cylindrical Deposit



REG. No.3760.D.D. 1939-350.

Rectangular Slab



Rectangular Slab

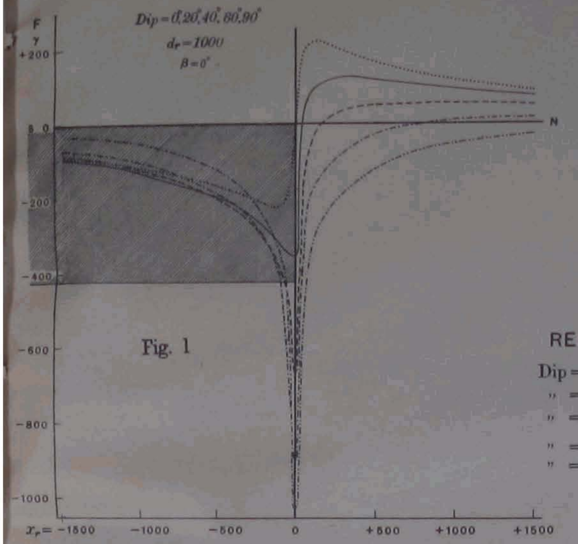


Fig. 1

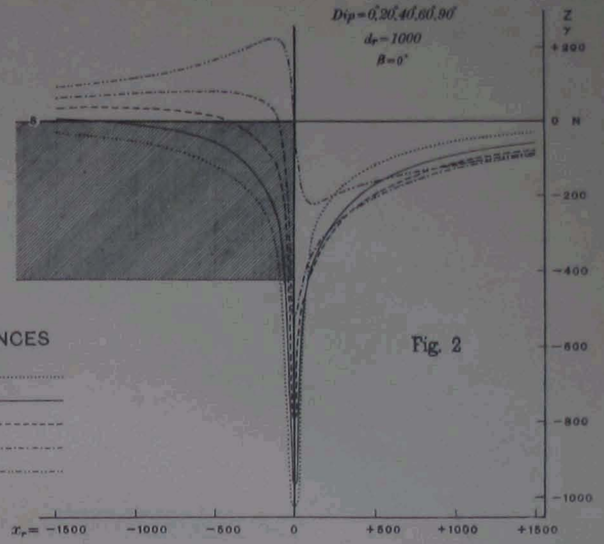


Fig. 2

REFERENCES

Dip = 0° ....
" = 20° ———
" = 40° – – – – –
" = 60° –·–·–·–
" = 90° –··–··–

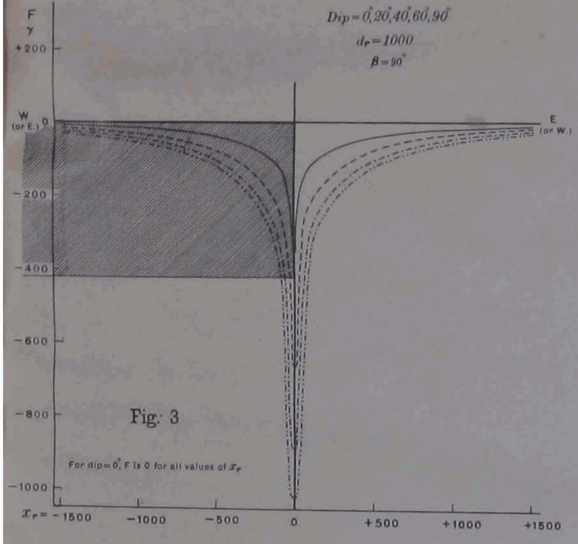


Fig. 3

For dip = 0°, F is 0 for all values of x_r

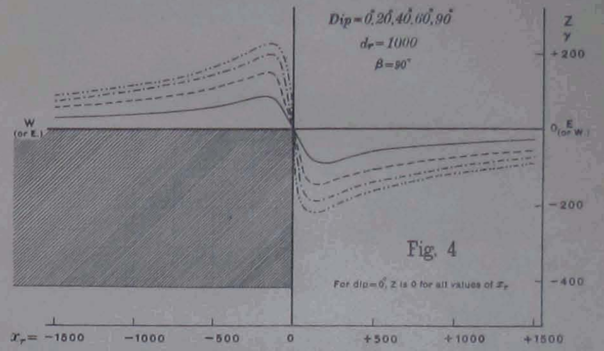


Fig. 4

For dip = 0°, Z is 0 for all values of x_r

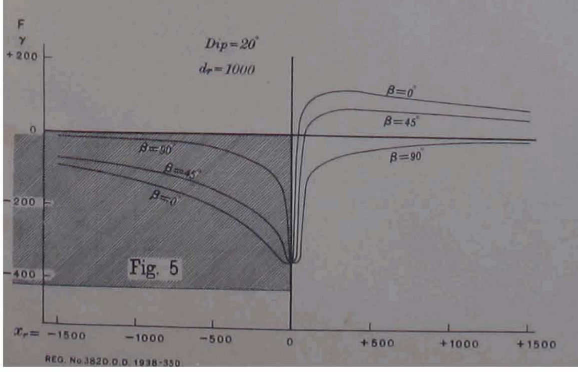


Fig. 5

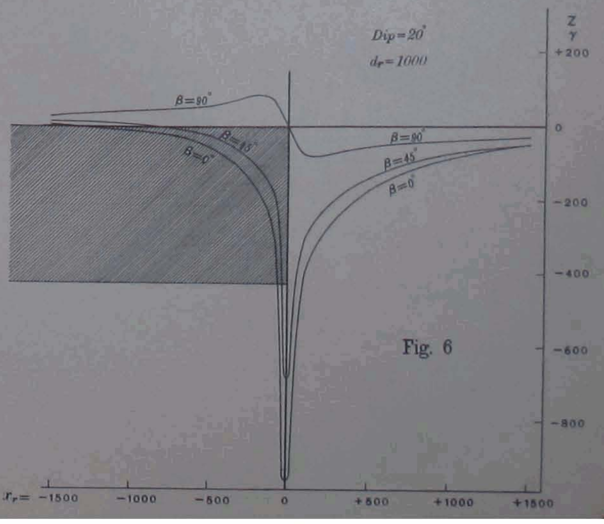


Fig. 6

REG. No. 3820 O.D. 1938-350

Simple Geological Fault

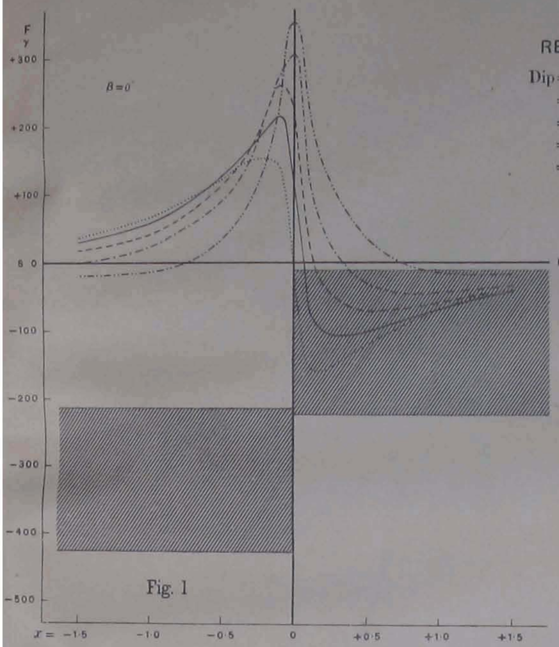


Fig. 1

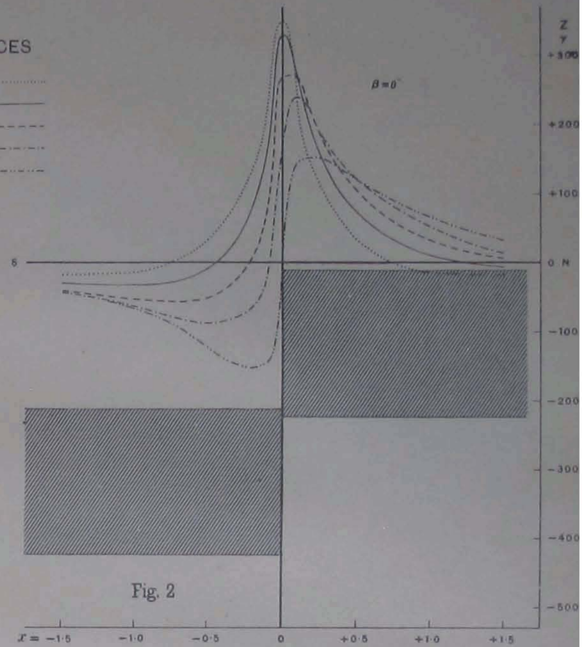


Fig. 2

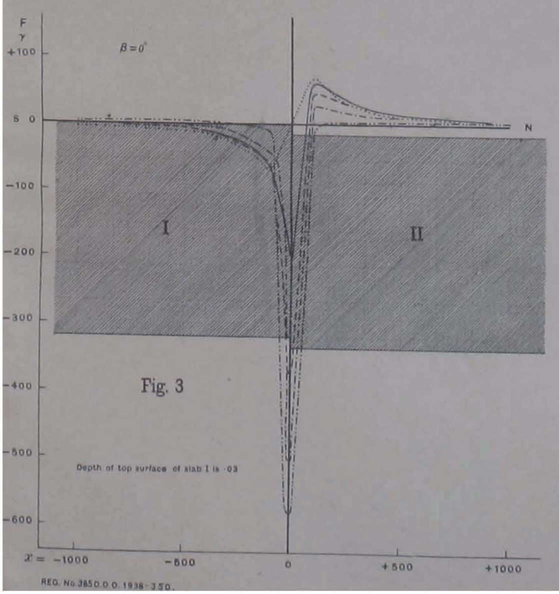


Fig. 3

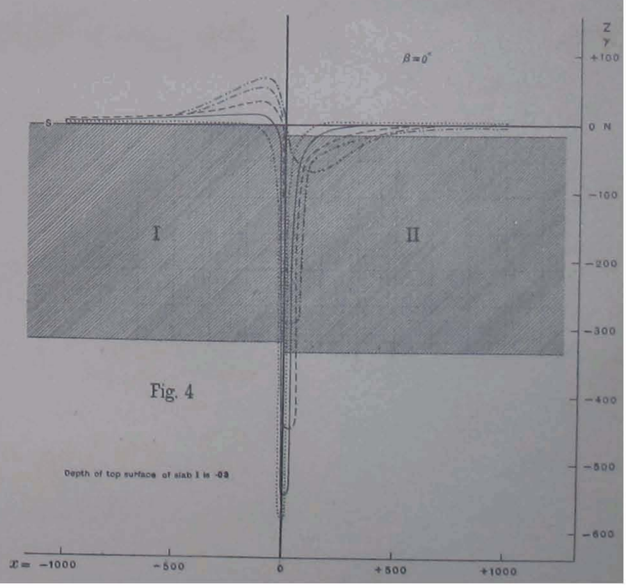


Fig. 4

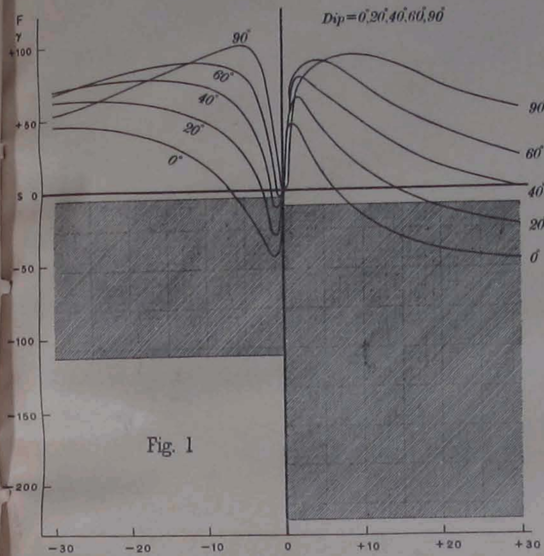


Fig. 1

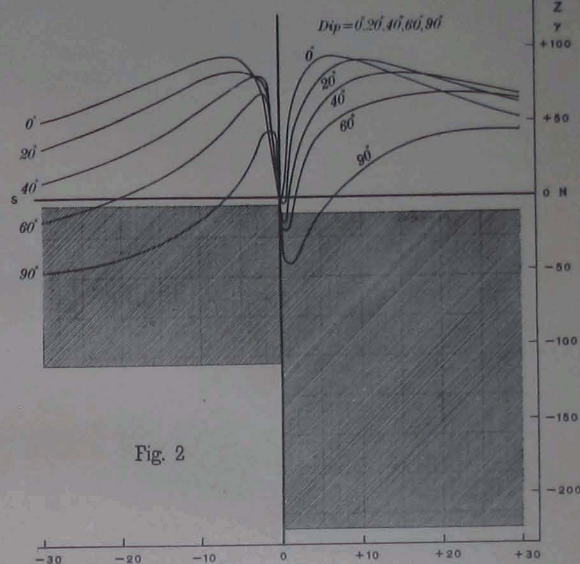


Fig. 2

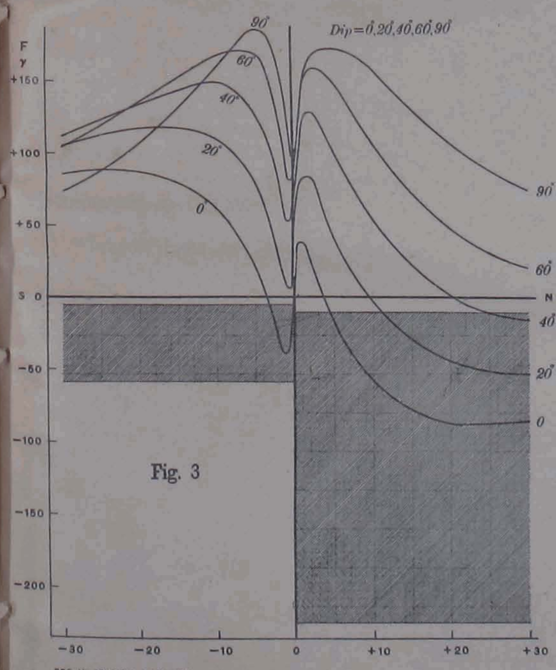


Fig. 3

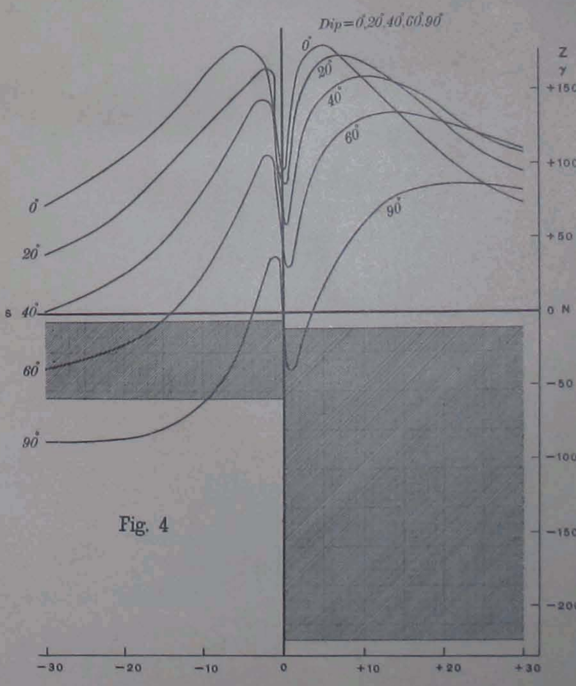


Fig. 4

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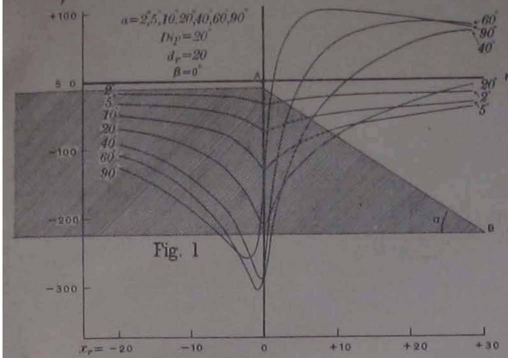

Fig. 1

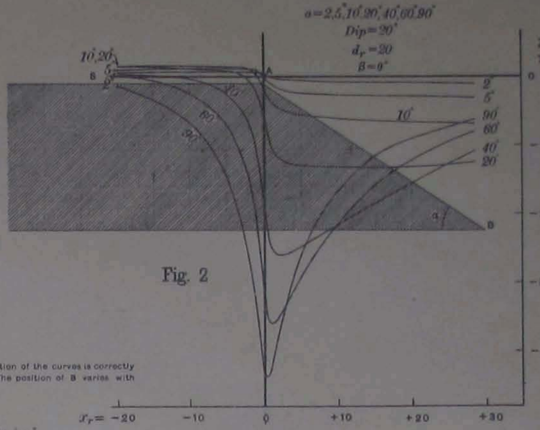

Fig. 2

In Figs 1 to 4, the position of the curves is correctly related to the point A. The position of B varies with the angle a

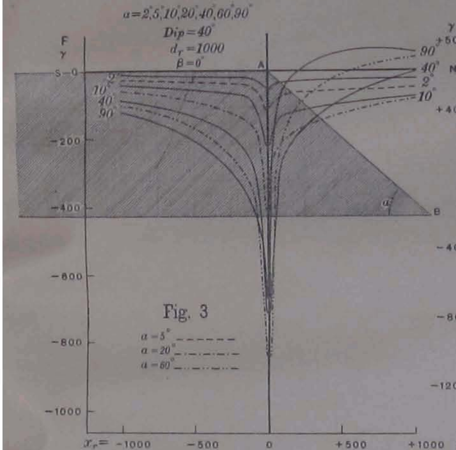

Fig. 3

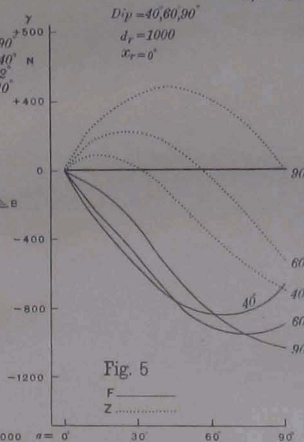

Fig. 5

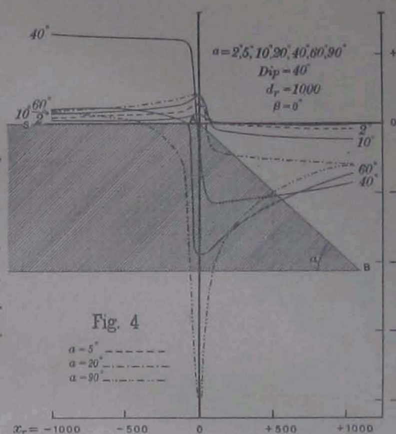

Fig. 4

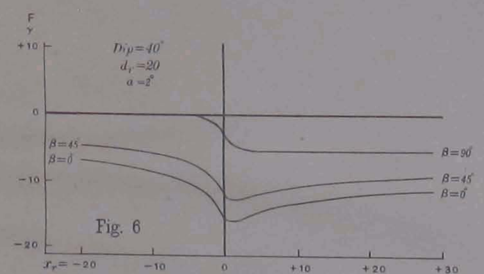

Fig. 6

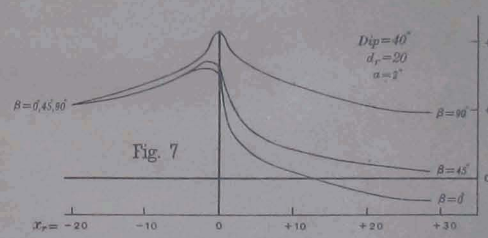

Fig. 7

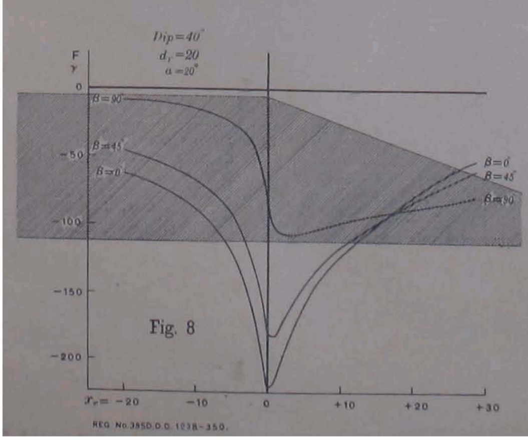

Fig. 8

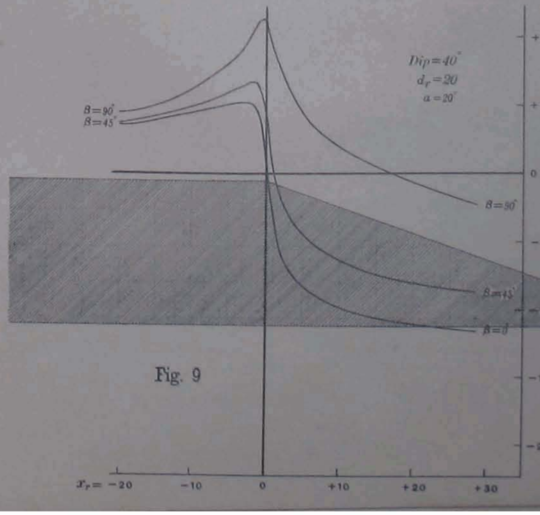

Fig. 9

Slab with inclined edge

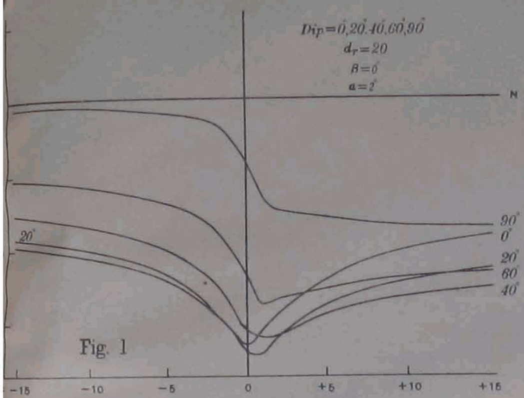

Fig. 1

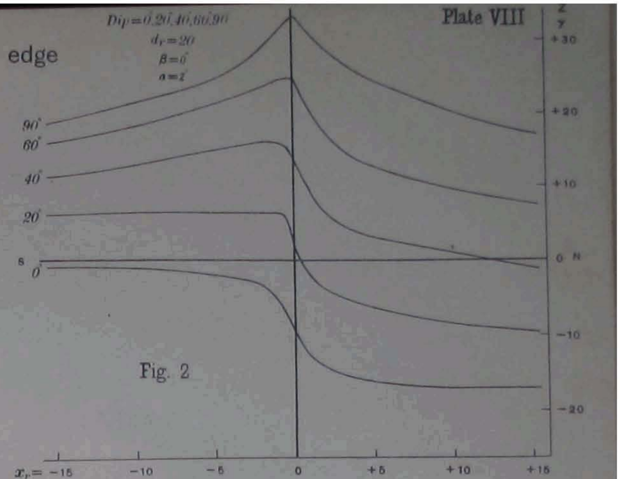

Fig. 2

Vertical Dyke

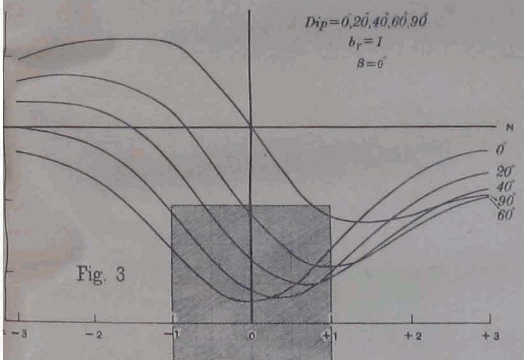

Fig. 3

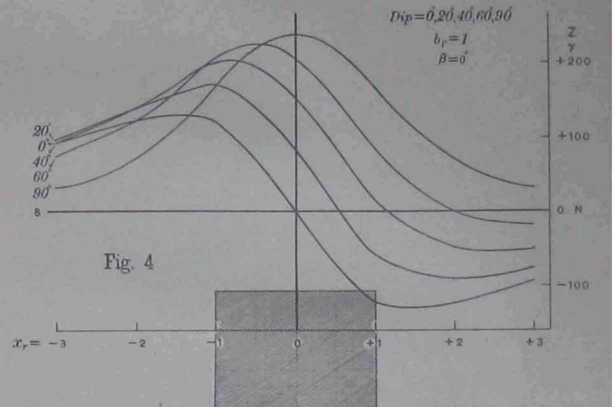

Fig. 4

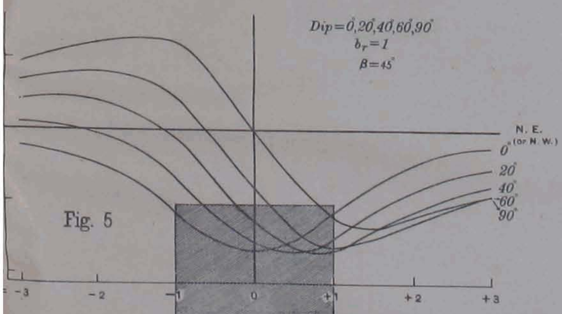

Fig. 5

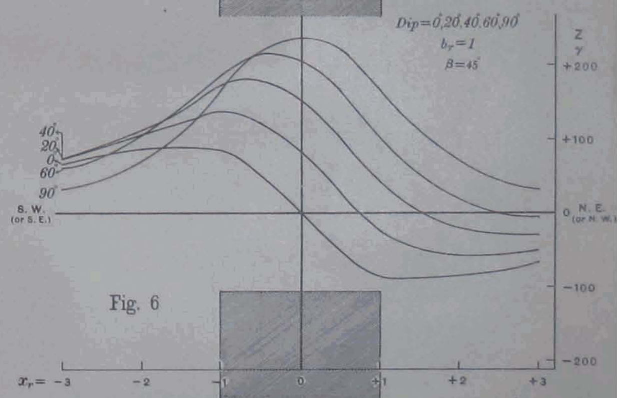

Fig. 6

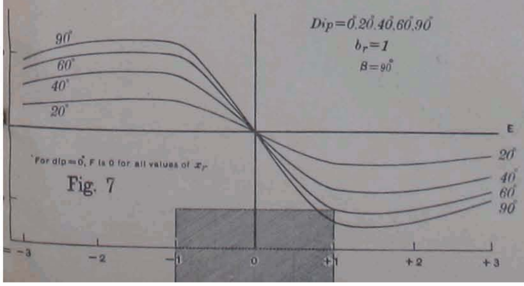

Fig. 7

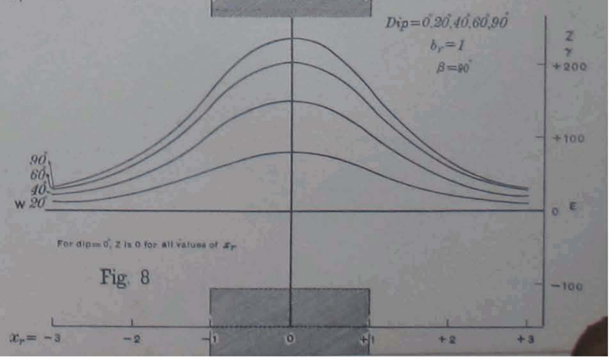

Fig. 8

Vertical Dyke

REFERENCES

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" = 60° -·-·-·-
" = 90° -··-··-

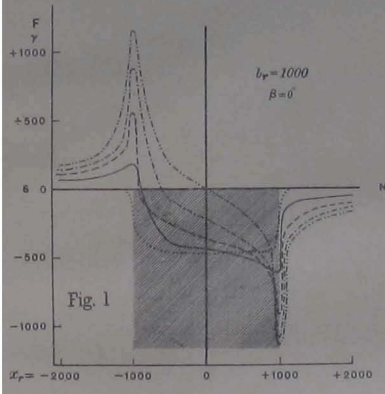

Fig. 1

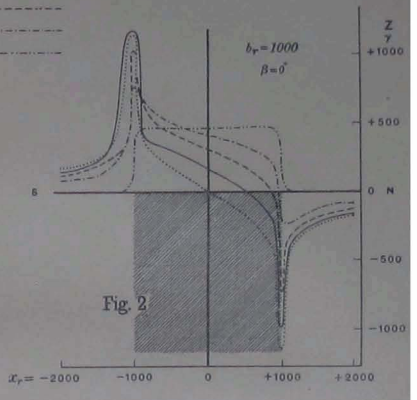

Fig. 2

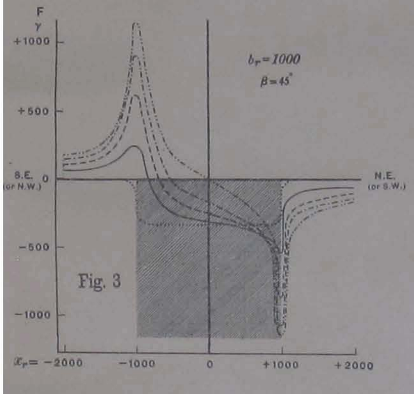

Fig. 3

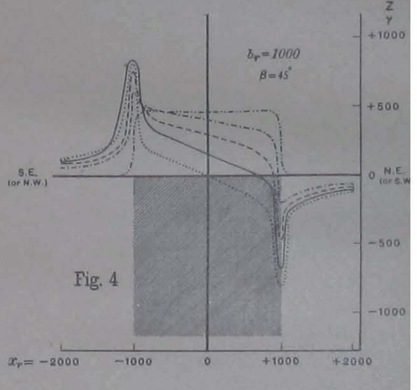

Fig. 4

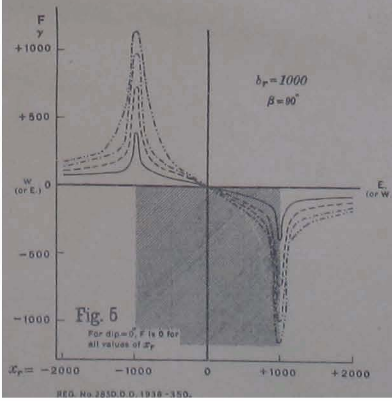

Fig. 5

For dip = 0°, F is 0 for all values of x_r

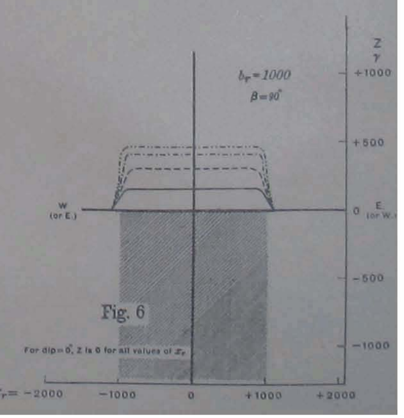

Fig. 6

For dip = 0°, Z is 0 for all values of x_r

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Horizontal Block

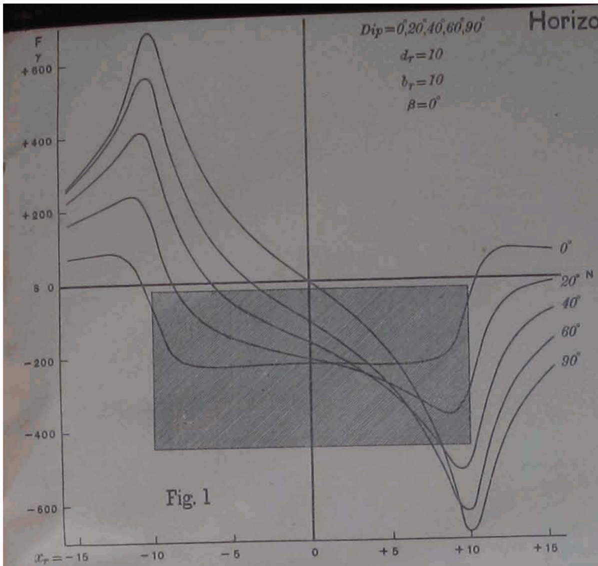


Fig. 1

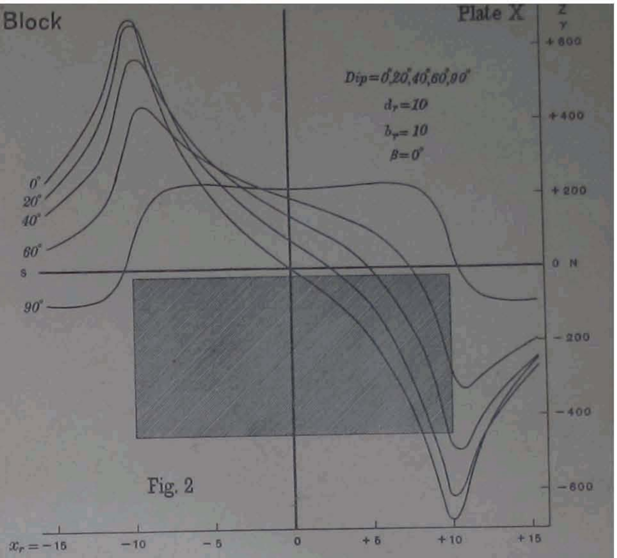


Fig. 2

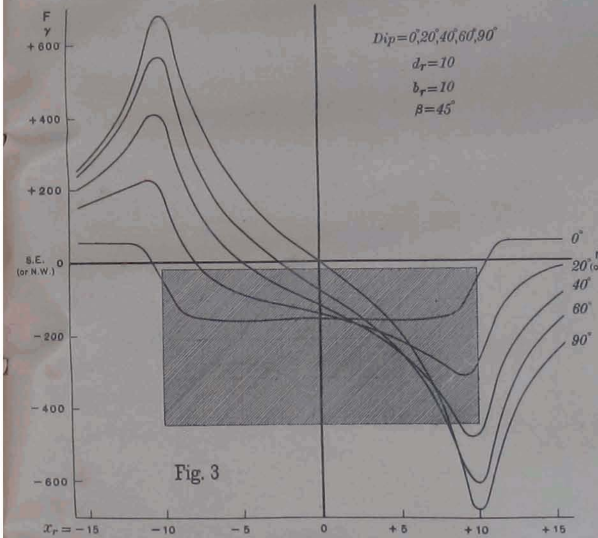


Fig. 3

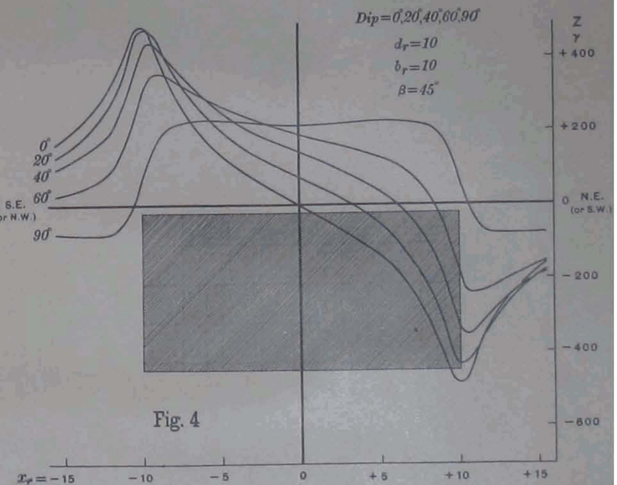


Fig. 4

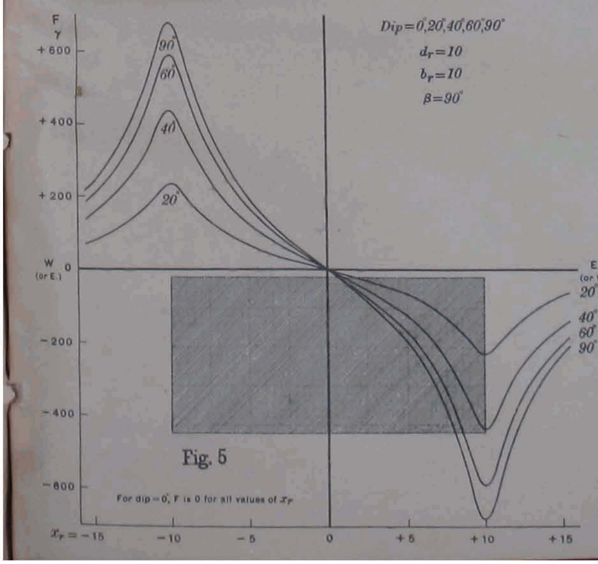


Fig. 5

For dip = 0°, F is 0 for all values of x_r

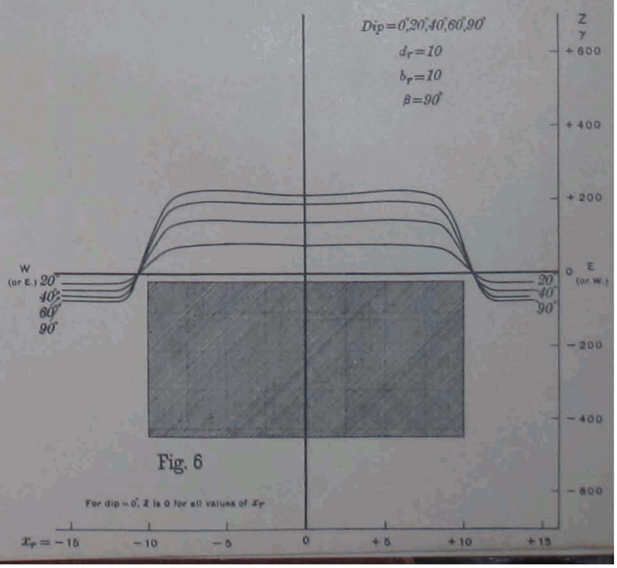
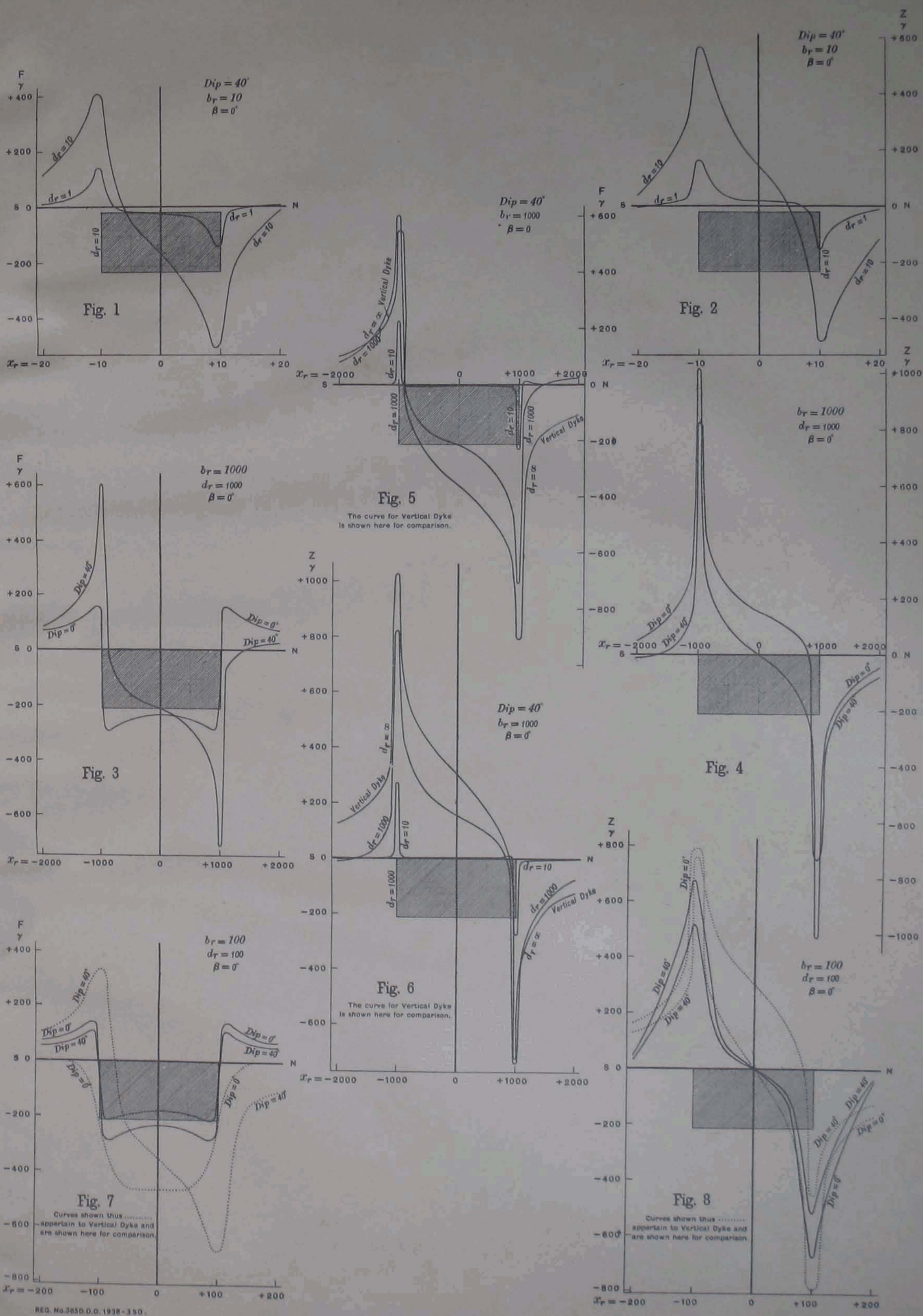


Fig. 6

For dip = 0°, Z is 0 for all values of x_r



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